Practice 6

Write down the Dantzig-Wolfe reformulation of the following LP:

$$\begin{aligned}
\min \quad & z = 2x_1 - x_2 + x_3 - x_4 \\
\text{s.t.} \quad & x_1 + 2x_2 \le 4 \\
& x_1 - x_2 \le 1 \\
& -5x_3 - x_4 \le -10 \\
& x_1 + 3x_2 - x_3 - 2x_4 \le 10 \\
& 2x_1 + x_2 \le 2 \\
& 2x_3 + x_4 \le 10 \\
& x_4 \le 5 \\
& x_i \ge 0, \ \forall i = 1, \ldots, 4
\end{aligned}$$

Solve the problem using column generation.

Restricted Master LP

$$z = \max \quad \mathbf{C}^1\left(\sum_{t=1}^{|\hat{\mathbf{T}}_1|}\lambda_t^1\mathbf{x}_t^1\right) + \mathbf{C}^2\left(\sum_{t=1}^{|\hat{\mathbf{T}}_2|}\lambda_t^2\mathbf{x}_t^2\right)$$

$$s.t.$$

$$\sum_{t=1}^{|\hat{\mathbf{T}}_1|}\lambda_t^1 = 1 \quad (dual:\pi_1)$$

$$\sum_{t=1}^{|\hat{\mathbf{T}}_2|}\lambda_t^2 = 1 \quad (dual:\pi_2)$$

$$\mathbf{A}^1\left(\sum_{t=1}^{|\hat{\mathbf{T}}_1|}\lambda_t^1\mathbf{x}_t^1\right) + \mathbf{A}^2\left(\sum_{t=1}^{|\hat{\mathbf{T}}_2|}\lambda_t^2\mathbf{x}_t^2\right) \le \mathbf{b} \quad (dual:\rho)$$

$$\lambda_t^1 \ge 0, \quad t = 1,\ldots,|\hat{\mathbf{T}}_1|$$

$$\lambda_t^2 \ge 0, \quad t = 1,\ldots,|\hat{\mathbf{T}}_2|$$

Pricing subproblem for x¹

$$\max \quad z_1 = \mathbf{C}^1\mathbf{x}^1 - \rho\mathbf{A}^1\mathbf{x}^1 - \pi_1$$

$$s.t.$$

$$\mathbf{D}^1\mathbf{x}^1 \le \mathbf{d}^1$$

$$\mathbf{x}^1 \ge 0$$

Pricing subproblem for x²

$$\max \quad z_2 = \mathbf{C}^2\mathbf{x}^2 - \rho\mathbf{A}^2\mathbf{x}^2 - \pi_2$$

$$s.t.$$

$$\mathbf{D}^2\mathbf{x}^2 \le \mathbf{d}^2$$

$$\mathbf{x}^2 \ge 0$$