Assignment 2:
Mixed Integer Programming
Branch-and-Bound and Gomory cuts

620-362 Applied Operations Research
Semester 2, 2008

Due Date: 5pm, 19th September 2008

Assignment prepared by Dr. Heng-Soon Gan

What to submit (hard copy, to the assignment boxes):
A report documenting necessary assumptions, workings, discussions and results for all parts of this assignment.
A signed copy of a plagiarism form.

What to submit (via email, to h.gan@ms.unimelb.edu.au):
NONE

Additional information:
This assignment will be marked out of 50.

Question 1 (25 marks)

A set of jobs must be processed on a single machine. The machine can only perform one job at a time. The time required to perform each job, the arrival date, the due date, and the penalty (in dollars) per day the job is late are given in the following table:

	Job Duration	Arrival Date	Due Date	Penalty
Job 1	4 days	Day 1	Day 3	4
Job 2	5 days	Day 0	Day 6	5
Job 3	3 days	Day 6	Day 12	7

A job can only start processing on or after the arrival date. Once a job starts, it has to be processed to completion (i.e. non-preemptable).

(a) **Formulate** a MIP to determine the job start times that will minimise the total penalty costs due to delayed jobs. The MIP must be written in standard mathematical notations. State all assumptions.

(b) Use **branch-and-bound** to solve the problem described in (a). You may use any method of your choice to solve the LPs (e.g. Xpress^{MP}, tutOR simplex engine etc.). Illustrate your answer using a branch-and-bound tree. Clearly indicate what you have branched on and give an index for your nodes (e.g. Node 3 means the 3rd node that you have explored). At each node, clearly indicate the lower and upper bounds, the solution obtained and the objective value.

BONUS SECTION (if attempted, a maximum of 10 additional marks will be awarded):

(c) Use the **Gomory procedure** to attempt to solve the problem described in (a) at the *root node*. Add *at most 5 Gomory cuts*. For the purpose of resolving your LP, use the *dual simplex* method. You only need to show the initial tableau (and the first pivot) and the final tableau at each iteration of the procedure. You may use any (primal/dual) simplex engine of your choice.

(d) Use **branch-and-cut** to solve the problem described in (a). Add at most two Gomory cuts at each node. *Do not* add Gomory cuts at the root node. Illustrate your answer using a branch-and-bound tree. Compare your results to that of (b), in terms of the total number of nodes searched and the maximum tree search depth.

Question 2 (25 marks)

Consider the following integer program:

(**IP**):
$$
\begin{aligned}
&\max \quad z = 5x_1 + 6x_2 + 3x_3 + 4x_4 \\
&s.t. \\
&3x_2 + x_3 + 2x_4 \le 3 \\
&2x_1 + 4x_2 + 2x_3 \le 10 \\
&x_1 + x_3 + 2x_4 \le 4.5 \\
&x_1, x_2, x_3, x_4 \quad \text{is integer}
\end{aligned}
$$

(a) Solve IP using the branch-and-bound method. You may use any method of your choice to solve the LPs (e.g. Xpress^{MP}, tutOR simplex engine etc.). Indicate the optimal solution to IP and illustrate your answer using a branch-and-bound tree. Clearly indicate what you have branched on and give an index for your nodes (e.g. Node 3 means the 3rd node that you have explored). At each node, clearly indicate the lower and upper bounds, the solution obtained and the objective value.

(b) Derive a Gomory cut for the relaxation of IP using the row corresponding to constraint $3x_2 + x_3 + 2x_4 \le 3$ in the simplex tableau. A relaxation of IP is to disregard all integrality requirements for variables x_i for i = 1,…,4.

(c) Append the Gomory derived in part (b) to IP and name it IP^{G1}. Solve IP^{G1} using the branch-and-bound method. Compare your results to that of part (a), in terms of the total number of nodes searched and the maximum tree search depth.